Lecture 5:
Instrumental Variables, 1 of 2

February 12, 2025

Course Administration

1. Hopefully you've turned in PS 2
2. PS 3 posted, due March 5
3. Lab after class next week
4. If you still need approval for your replication paper, or need to choose a new one, do it ASAP
5. March 19: quantitative progress report due
6. Let's divide next week's articles
7. Please come see me about your replication paper
8. Any other issues?

Plan for Today

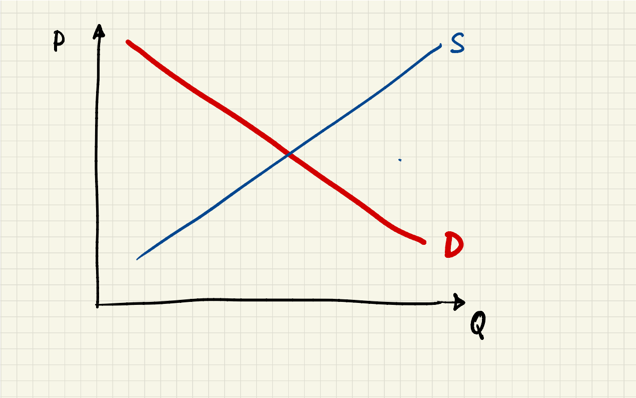
1. IV Overview
2. A&K: Oldie but Goodie

IV Background

1. Origins and motivation of IV
2. More general formulation
3. Regression framework: Wald estimate and 2SLS
4. Testing assumptions underlying IV

IV Origins

IV Origin Story



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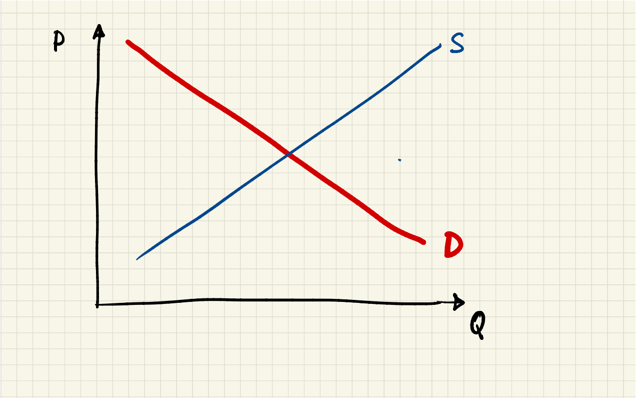

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$$Q_t^D = \alpha P_t + \epsilon_t$$

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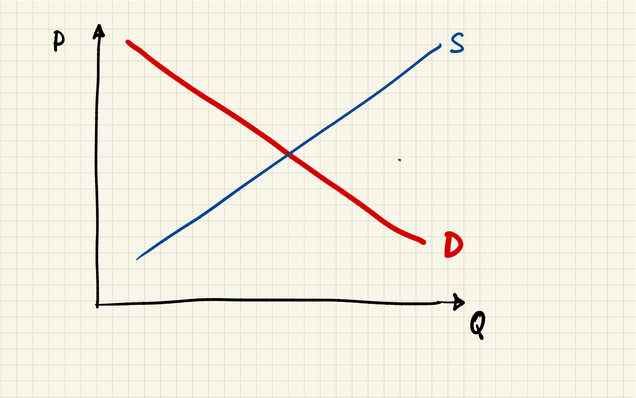

- We want to know the impact of price on quantity demanded, or

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- But we only observe equilibrium where $S = D$

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- When might this not be the case?
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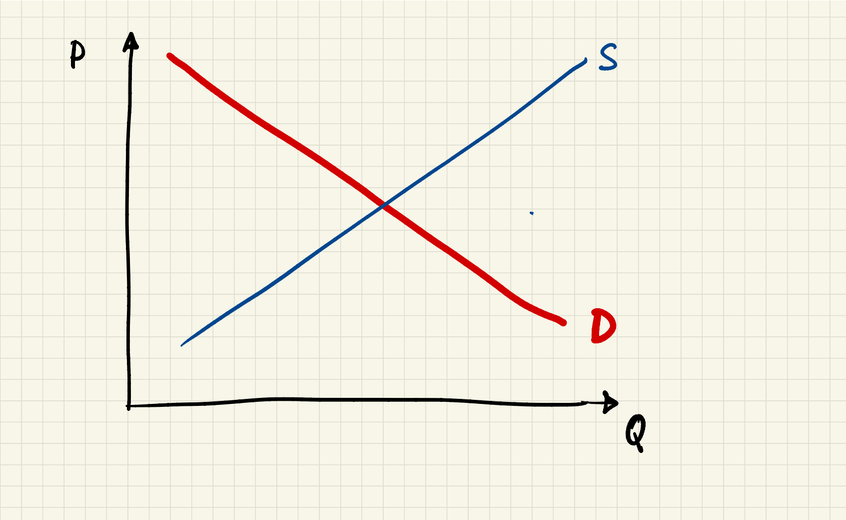
  - P increases and Q increases – does increase in P cause increase in Q? no!
  - an omitted variable – new info – causes both P and Q^D to increase

Potential Solutions to this Problem

We need something that shifts P AND does not affect Q_D



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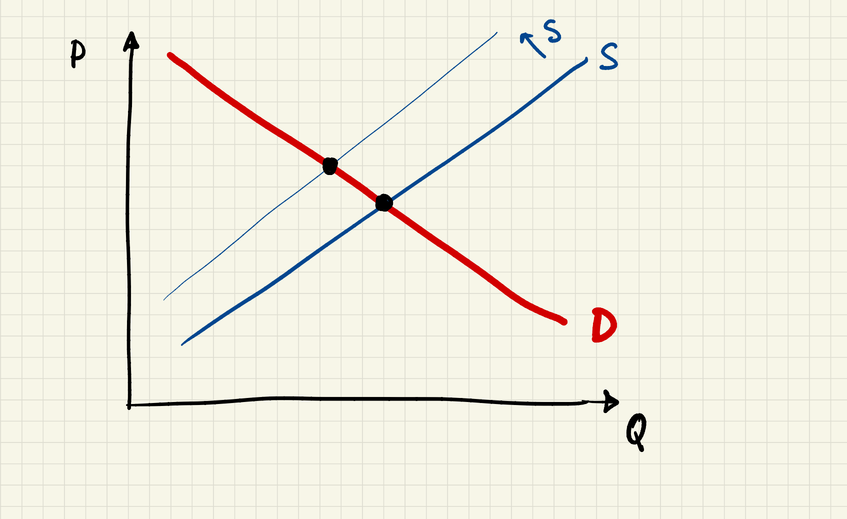

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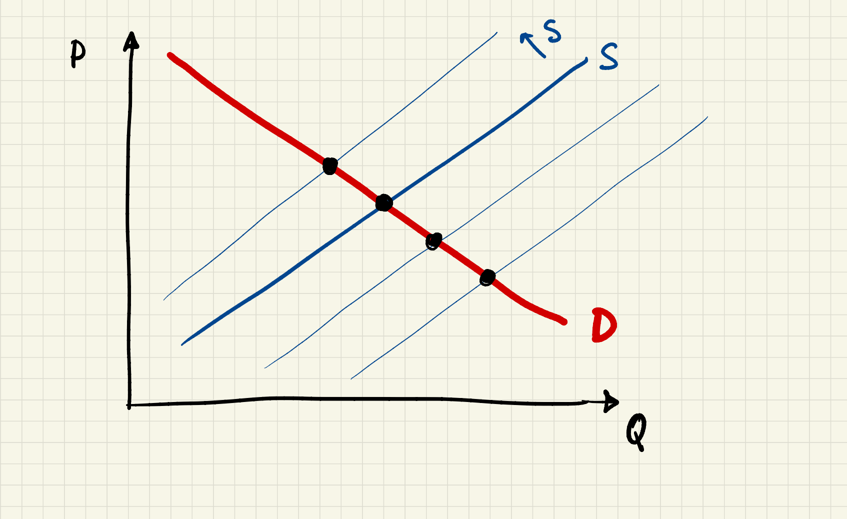

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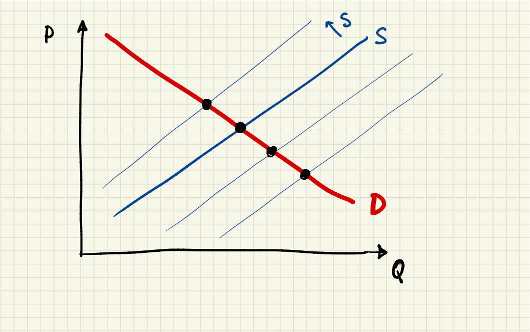

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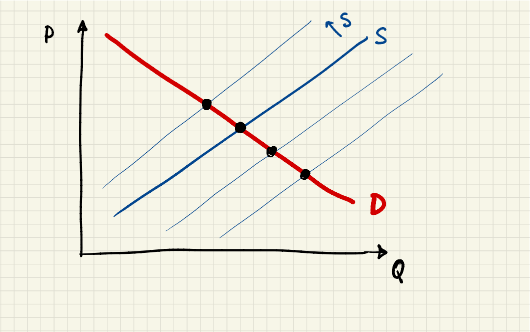

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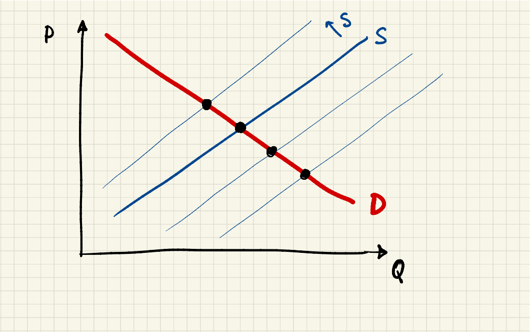

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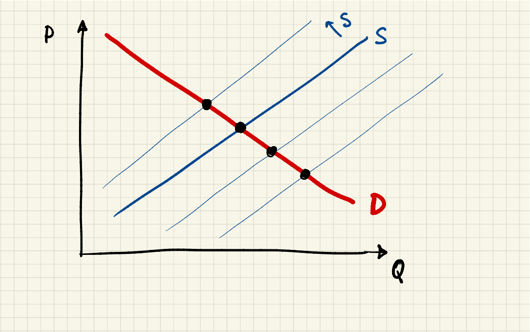

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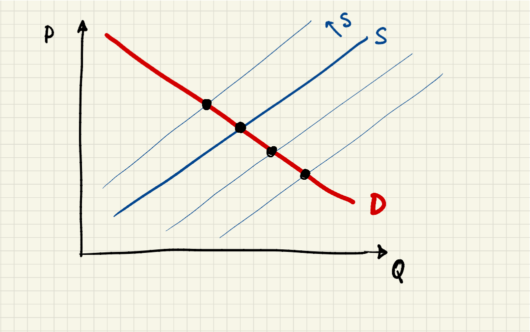

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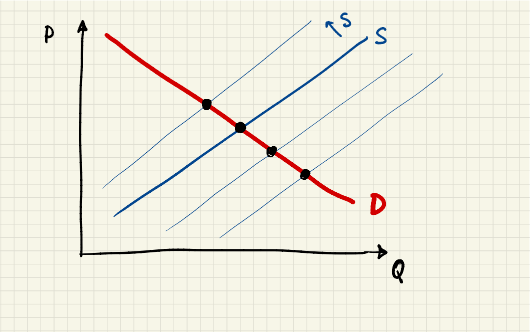

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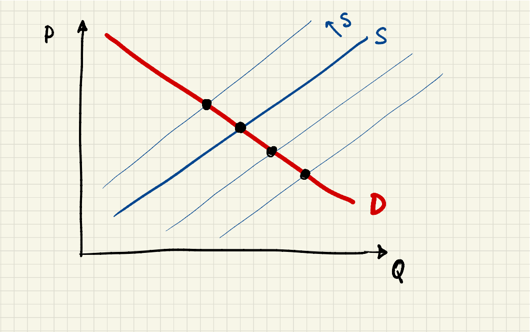

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We need something that generates variation in P unrelated to outcome Q_D

IV: More General Problem

General Formulation of Basic Problem

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- Problem: We don't observe A
- Are we stuck?

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- instrument for A and K is quarter of birth
- instrument for Angrist et al is lottery winning

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Instead of full variation in X, use variation in X that comes from Z

Conditions for an instrument, Z

1. Instrument must be correlated with endogenous variable:

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- what if $\hat{\gamma} = 1$? Just use Z directly – and your instrument may be crap
- Note that $\hat{\lambda} = \hat{\beta}\hat{\gamma}$
- this means that

$$\hat{\beta} = \frac{\hat{\lambda}}{\hat{\gamma}} = \frac{\text{cov}(Y, Z)/\text{var}(Z)}{\text{cov}(X, Z)/\text{var}(Z)} = \frac{\text{cov}(Y, Z)}{\text{cov}(X, Z)}$$

- in words, how much of the change in Y due to Z is explained by the change in X due to Z

Simplifying with a Binary Instrument

- Let's think of Z as being binary: won lottery or not
- Write the Wald Estimate as

$$\hat{\beta}_{IV} = \frac{\text{cov}(Y, Z)}{\text{cov}(X, Z)} = \frac{E(Y|Z=1) - E(Y|Z=0)}{E(X|Z=1) - E(X|Z=0)}$$

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- In words? mean difference in test scores for students who won KIPP lottery and those who didn't divided by mean difference in KIPP attendance between those who won KIPP lottery and those who didn't
- if the instrument has no effect the denominator goes to zero (!!)
- you can do this with means or a regression

Implementing More General IV

- first stage: $X = \gamma Z + \delta, \rightarrow \hat{X}$
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- note that you can do this in two steps – that's why it's called two-stage

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Note that β_{IV} is biased in finite samples, but consistent – β_{OLS} is both unbiased and consistent

IV: Testing Underlying Assumptions

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  1. correlation between instrument and things that should not be affected by treatment
     – recall that instrument works *only* through treatment
  2. relationship betwen Z and Y where Z should not impact treatment
- Section III in A and K is an attempt to do these things
- Any credible IV paper should some argument along these lines

Angrist and Kreuger

Second Half of Lecture 5

1. Why do we read this old paper?
2. Causal question and endogeneity concerns
3. Data
4. Instruments and validity
5. Results
6. Reflection, many years on

1. Why do we read this old paper?

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- Not quite right, so there's room to think

2. Causal question and endogeneity

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Identification issues

- why don't we just estimate

$$\text{income}_i = \beta_0 + \beta_1 \text{schooling}_i + \beta_2 X_i + \epsilon_i$$

?

3. Data

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- Cross-section or panel? cross-section
- What's the unit of observation? person (not in a year)

3. Instruments

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What are the instruments?

- generally, quarter of birth
- what's the story about how this works?
- the story is that this works through compulsory schooling laws + different age of starting school, aka quarter of birth
- See Eq. 1, p. 997: quarter*year (10 years), so 40 instruments
- or 50 states * 4 quarters + 10 years * 4 quarters = 240 instruments

Showing the Easy IV Condition: $\text{corr}(X, Z) \neq 0$

IV condition No. 1: $\text{corr}(X, Z) \neq 0$. What evidence do they offer?

- First, the story linking quarter of birth to compulsory schooling laws

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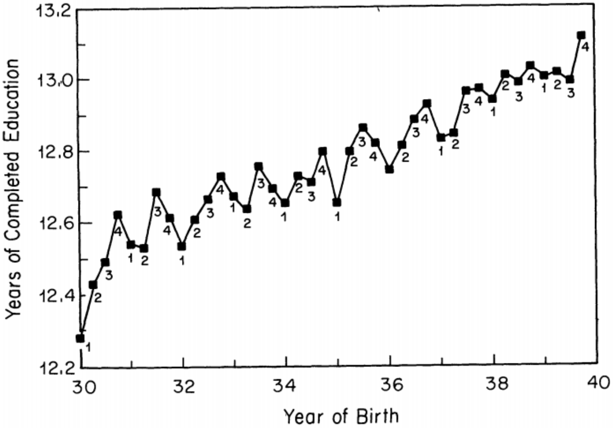
- Figures 1-3, first part of Table 1, and Figure 4 (which is Figures 1 to 3 in differences)
- and we have to believe that the diff-in-diff with compulsory schooling laws is not endogenous
- note that Section 1 is mostly dedicated to this

Figure 1: Quarter of Birth and Education



Data from 1980 Census, ages 50 (left) to 40 (right)

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  - QOB matters for attendance at age 16 when school leaving age is 16, but not when school leaving age is 17 or 18 – suggesting that the law is the mechanism

TABLE I
THE EFFECT OF QUARTER OF BIRTH ON VARIOUS EDUCATIONAL OUTCOME VARIABLES

Outcome variable	Birth cohort	Mean	Quarter-of-birth effect[a] I	II	III	F-test[b] [P-value]
Total years of education	1930–1939	12.79	−0.124	−0.086	−0.015	24.9
			(0.017)	(0.017)	(0.016)	[0.0001]
	1940–1949	13.56	−0.085	−0.035	−0.017	18.6
			(0.012)	(0.012)	(0.011)	[0.0001]
High school graduate	1930–1939	0.77	−0.019	−0.020	−0.004	46.4
			(0.002)	(0.002)	(0.002)	[0.0001]
	1940–1949	0.86	−0.015	−0.012	−0.002	54.4
			(0.001)	(0.001)	(0.001)	[0.0001]
Years of educ. for high school graduates	1930–1939	13.99	−0.004	0.051	0.012	5.9
			(0.014)	(0.014)	(0.014)	[0.0006]
	1940–1949	14.28	0.005	0.043	−0.003	7.8
			(0.011)	(0.011)	(0.010)	[0.0017]
College graduate	1930–1939	0.24	−0.005	0.003	0.002	5.0
			(0.002)	(0.002)	(0.002)	[0.0021]
	1940–1949	0.30	−0.003	0.004	0.000	5.0
			(0.002)	(0.002)	(0.002)	[0.0018]
Completed master's degree	1930–1939	0.09	−0.001	0.002	−0.001	1.7
			(0.001)	(0.001)	(0.001)	[0.1599]
	1940–1949	0.11	0.000	0.004	0.001	3.9
			(0.001)	(0.001)	(0.001)	[0.0091]
Completed doctoral degree	1930–1939	0.03	0.002	0.003	0.000	2.9
			(0.001)	(0.001)	(0.001)	[0.0332]
	1940–1949	0.04	−0.002	0.001	−0.001	4.3
			(0.001)	(0.001)	(0.001)	[0.0050]

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TABLE II
PERCENTAGE OF AGE GROUP ENROLLED IN SCHOOL BY BIRTHDAY AND LEGAL DROPOUT AGE[a]

	Type of state law[b]		
Date of birth	School-leaving age: 16 (1)	School-leaving age: 17 or 18 (2)	Column (1) − (2)
	Percent enrolled April 1, 1960		
1. Jan 1–Mar 31, 1944	87.6	91.0	−3.4
(age 16)	(0.6)	(0.9)	(1.1)
2. Apr 1–Dec 31, 1944	92.1	91.6	0.5
(age 15)	(0.3)	(0.5)	(0.6)
3. Within-state diff.	−4.5	−0.6	−4.0
(row 1 − row 2)	(0.7)	(1.0)	(1.2)

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- What if people who are born in the first quarter are more likely to be mentally ill? should this challenge their estimation strategy?
  - no, if the effect works through education
  - yes, if it works through pathways other than education

4. Results

Interpreting the Results: Wald Estimate

OLS estimate:

$$\hat{\beta}_{OLS} = \frac{\Delta Y}{\Delta X}$$

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TABLE III

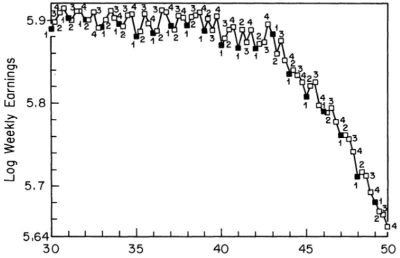
PANEL A: WALD ESTIMATES FOR 1970 CENSUS—MEN BORN 1920–1929[a]

	(1) Born in 1st quarter of year	(2) Born in 2nd, 3rd, or 4th quarter of year	(3) Difference (std. error) (1) − (2)
ln (wkly. wage)	5.1484	5.1574	−0.00898 (0.00301)
Education	11.3996	11.5252	−0.1256 (0.0155)
Wald est. of return to education			0.0715 (0.0219)
OLS return to education[b]			0.0801 (0.0004)

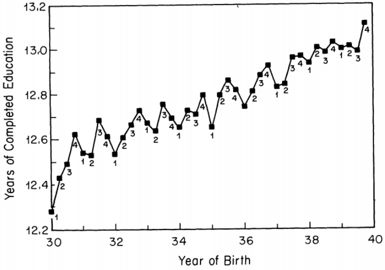
Visual Wald Estimate

Divide changes due to QOB in Figure 5 by changes due to QOB in Figure 1

Numerator: ΔY due to ΔZ



Denominator: ΔX due to ΔZ



Interpreting the Results: OLS and IV

- Interpret OLS coefficient
- Interpret IV coefficient

TABLE IV
OLS AND TSLS ESTIMATES OF THE RETURN TO EDUCATION FOR MEN BORN 1920–1929: 1970 CENSUS[a]

Independent variable	(1) OLS	(2) TSLS	(3) OLS	(4) TSLS	(5) OLS	(6) TSLS	(7) OLS	(8) TSLS
Years of education	0.0802	0.0769	0.0802	0.1310	0.0701	0.0669	0.0701	0.1007
	(0.0004)	(0.0150)	(0.0004)	(0.0334)	(0.0004)	(0.0151)	(0.0004)	(0.0334)
Race (1 = black)	—	—	—	—	0.2980	−0.3055	−0.2980	−0.2271
					(0.0043)	(0.0353)	(0.0043)	(0.0776)
SMSA (1 = center city)	—	—	—	—	0.1343	0.1362	0.1343	0.1163
					(0.0026)	(0.0092)	(0.0026)	(0.0198)
Married (1 = married)	—	—	—	—	0.2928	0.2941	0.2928	0.2804
					(0.0037)	(0.0072)	(0.0037)	(0.0141)
9 Year-of-birth dummies	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
8 Region of residence dummies	No	No	No	No	Yes	Yes	Yes	Yes
Age	—	—	0.1446	0.1409	—	—	0.1162	0.1170
			(0.0676)	(0.0704)			(0.0652)	(0.0662)
Age-squared	—	—	−0.0015	−0.0014	—	—	−0.0013	−0.0012
			(0.0007)	(0.0008)			(0.0007)	(0.0007)
χ^2 [dof]	—	36.0 [29]	—	25.6 [27]	—	34.2 [29]	—	28.8 [27]

5. A and K, 30 Years On

A and K, 30 Years On

- Compulsory schooling part still has traction
- Quarter of birth part not as much
- Subject to a scathing critique (BBJ, 1995), but method much copied
  - quarter of birth impacts school performance
  - health differences by quarter of birth
  - regional patterns in quarter of birth
- Next class: we'll discuss problems of using many and weak instruments

Next Lecture

- Read
  - the instrument paper to which you're assigned
  - trade if you'd like
  - skim the intro of the other
- Summary due next week if you're on the list