Lecture 5:
Instrumental Variables, 1 of 2

October 4, 2023

Course Administration

1. Hopefully you've turned in PS 2
2. PS 3 posted, due October 25
3. If you still need approval for your replication paper, or need to choose a new one, do it ASAP
4. Lectures tab cleaned up
5. Nov. 1: quantitative progress report due
6. Let's divide next week's articles
7. Please come see me about your replication paper
8. Any other issues?

Plan for Today

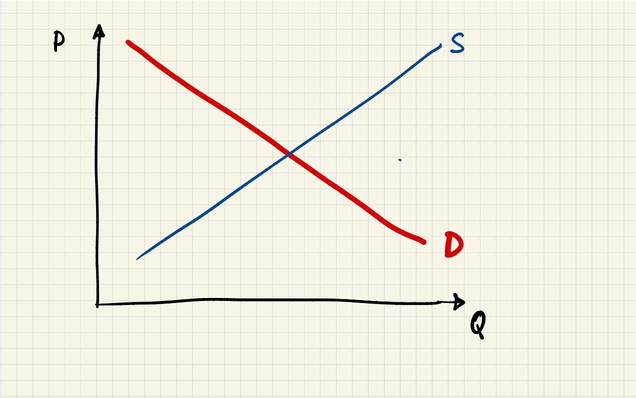
1. IV Overview
2. A&K: Oldie but Goodie

IV Background

1. Origins and motivation of IV
2. More general formulation
3. IV as directed acyclical graph
4. Regression framework: Wald estimate and 2SLS
5. Testing assumptions underlying IV

IV Origins

IV Origin Story



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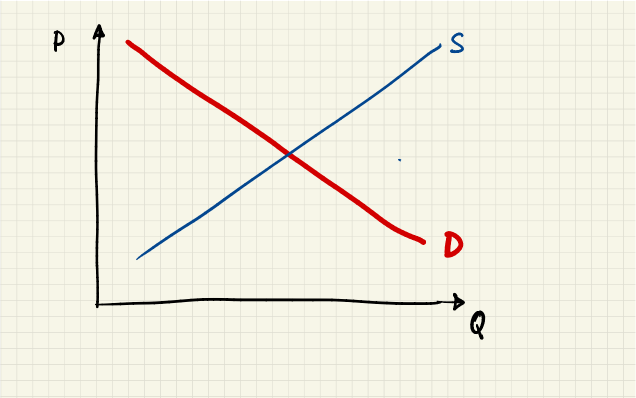

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$$Q_t^D = \alpha P_t + \epsilon_t$$

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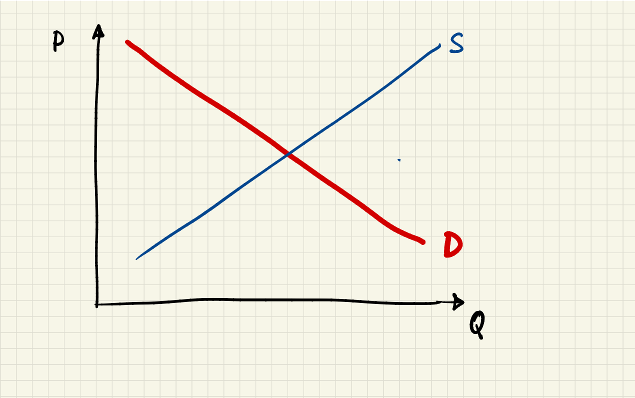

- We want to know the impact of price on quantity demanded, or

$$Q_t^D = \alpha P_t + \epsilon_t$$

- But we only observe equilibrium where $S = D$

What's the Trouble with this Estimation?

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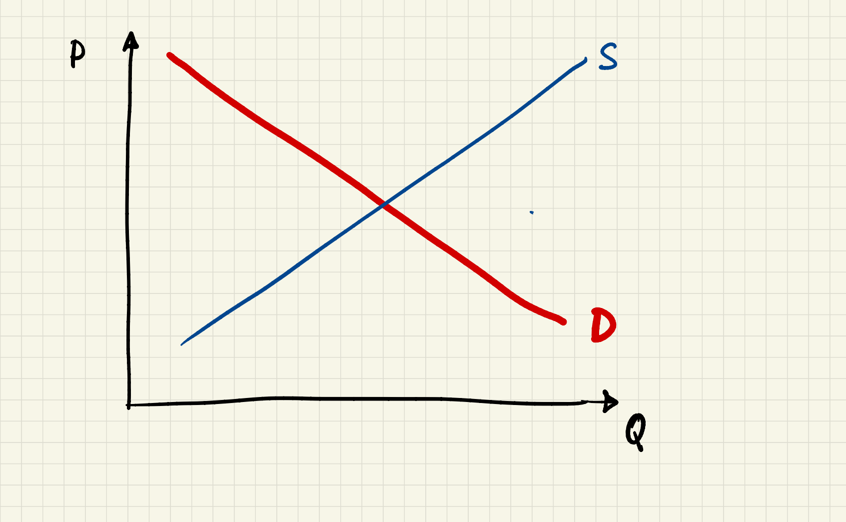
- When might this not be the case?
  - suppose a study shows that chocolate is good for your health
  - P increases and Q increases – does increase in P cause increase in Q? no!
  - an omitted variable – new info – causes both P and Q^D to increase

Potential Solutions to this Problem

We need something that shifts P AND does not affect Q_D



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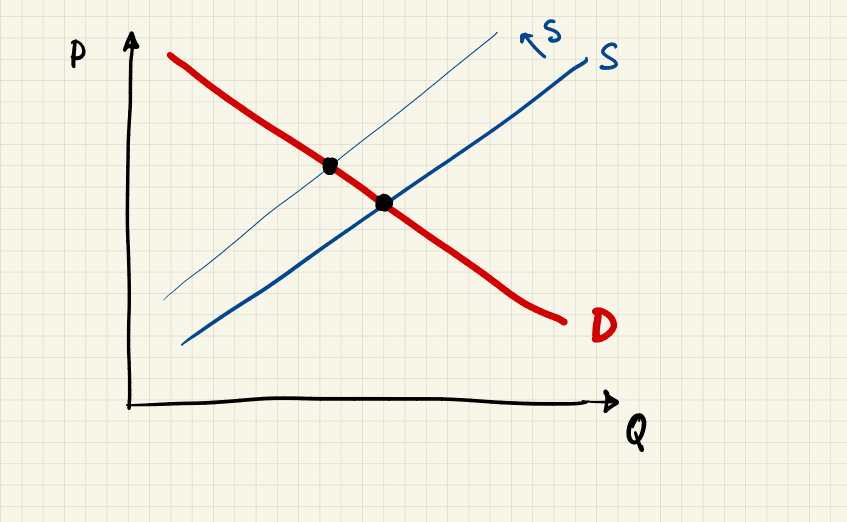

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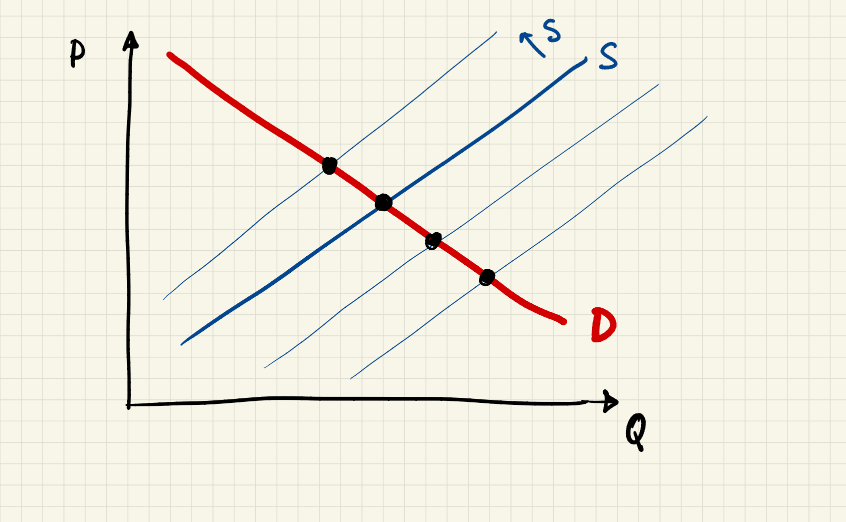

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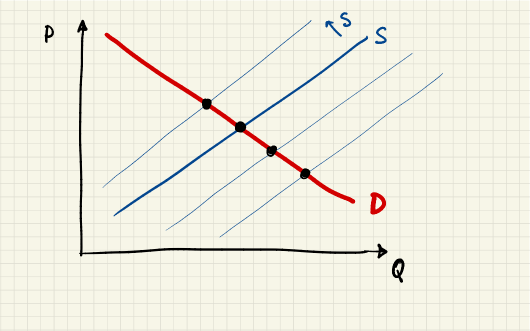

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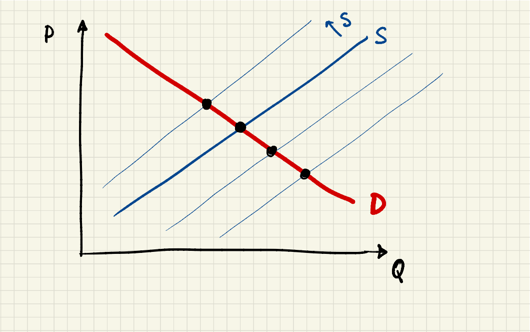

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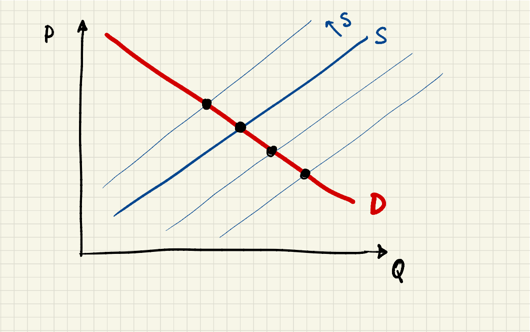

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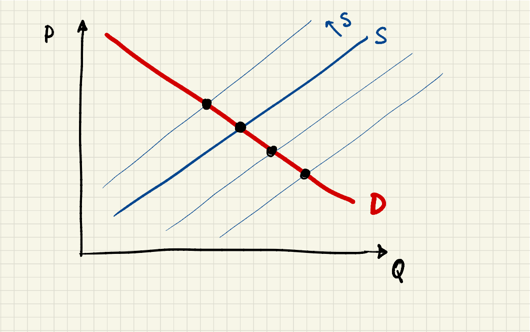

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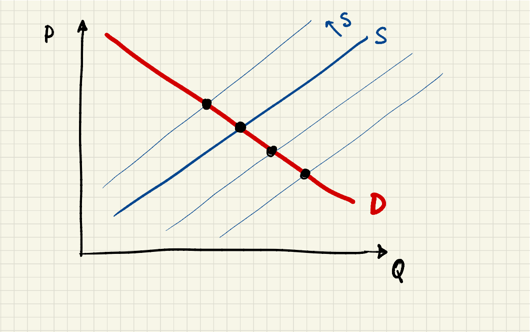

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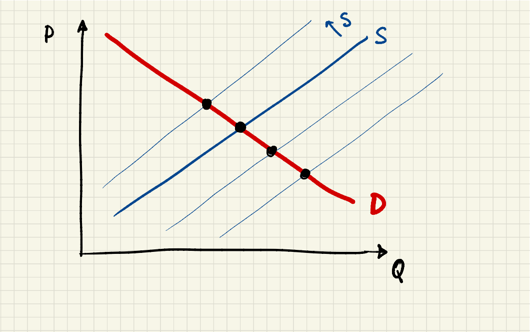

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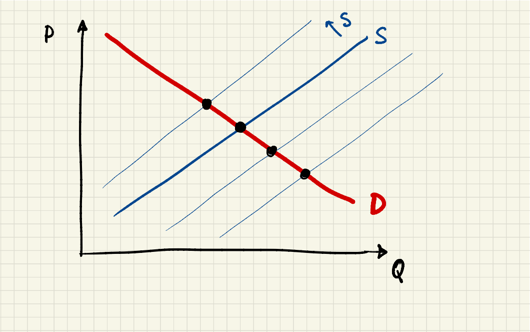

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We need something that generates variation in P unrelated to outcome Q_D

IV: More General Problem

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$$Y = \beta X + \nu$$

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- Problem: We don't observe A
- Are we stuck?

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- instrument for A and K is quarter of birth
- instrument for Angrist et al is lottery winning

IV: Regression Framework

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Instead of full variation in X, use variation in X that comes from Z

Conditions for an instrument, Z

1. Instrument must be correlated with endogenous variable:

$$\text{cov}(X, Z) \neq 0$$

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- what if $\hat{\gamma} = 1$? Just use Z directly – and your instrument may be crap
- Note that $\hat{\lambda} = \hat{\beta}\hat{\gamma}$
- this means that

$$\hat{\beta} = \frac{\hat{\lambda}}{\hat{\gamma}} = \frac{\text{cov}(Y, Z)/\text{var}(Z)}{\text{cov}(X, Z)/\text{var}(Z)} = \frac{\text{cov}(Y, Z)}{\text{cov}(X, Z)}$$

- in words, how much of the change in Y due to Z is explained by the change in X due to Z

Simplifying with a Binary Instrument

- Let's think of Z as being binary: won lottery or not
- Write the Wald Estimate as

$$\hat{\beta}_{IV} = \frac{\mathrm{cov}(Y, Z)}{\mathrm{cov}(X, Z)} = \frac{E(Y|Z=1) - E(Y|Z=0)}{E(X|Z=1) - E(X|Z=0)}$$

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- In words? mean difference in test scores for students who won KIPP lottery and those who didn't divided by mean difference in KIPP attendance between those who won KIPP lottery and those who didn't
- if the instrument has no effect the denominator goes to zero (!!)
- you can do this with means or a regression

Implementing More General IV

- first stage: $X = \gamma Z + \delta, \rightarrow \hat{X}$
- second stage: $Y = \beta \hat{X} + \nu = \beta(\hat{\gamma} Z) + \nu$
- note that you can do this in two steps – that's why it's called two-stage

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Note that β_{IV} is biased in finite samples, but consistent – β_{OLS} is both unbiased and consistent

IV: Testing Underlying Assumptions

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  1. correlation between instrument and things that should not be affected by treatment
     – recall that instrument works *only* through treatment
  2. relationship betwen Z and Y where Z should not impact treatment
- Section III in A and K is an attempt to do these things
- Any credible IV paper should some argument along these lines

Angrist and Kreuger

Second Half of Lecture 5

1. Why do we read this old paper?
2. Causal question and endogeneity concerns
3. Data
4. Instruments and validity
5. Results
6. Reflection, many years on

1. Why do we read this old paper?

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- Not quite right, so there's room to think

2. Causal question and endogeneity

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Causal question?

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Identification issues

- why don't we just estimate

$$\text{income}_i = \beta_0 + \beta_1 \text{schooling}_i + \beta_2 X_i + \epsilon_i$$

?

3. Data

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- 1960, 1970 and 1980 5 percent samples from Census
- Cross-section or panel? cross-section
- What's the unit of observation? person (not in a year)

3. Instruments

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- generally, quarter of birth
- what's the story about how this works?
- the story is that this works through compulsory schooling laws + different age of starting school, aka quarter of birth

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- generally, quarter of birth
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- or 50 states * 4 quarters + 10 years * 4 quarters = 240 instruments

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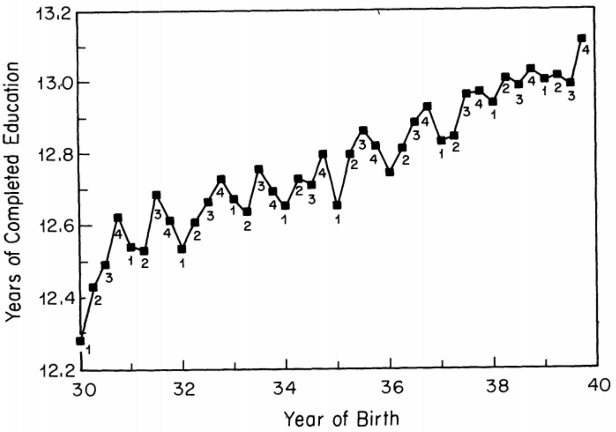
- Figures 1-3, first part of Table 1, and Figure 4 (which is Figures 1 to 3 in differences)
- and we have to believe that the diff-in-diff with compulsory schooling laws is not endogenous
- note that Section 1 is mostly dedicated to this

Figure 1: Quarter of Birth and Education



Data from 1980 Census, ages 50 (left) to 40 (right)

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TABLE I
THE EFFECT OF QUARTER OF BIRTH ON VARIOUS EDUCATIONAL OUTCOME VARIABLES

Outcome variable	Birth cohort	Mean	Quarter-of-birth effect[a] I	II	III	F-test[b] [P-value]
Total years of education	1930–1939	12.79	−0.124	−0.086	−0.015	24.9
			(0.017)	(0.017)	(0.016)	[0.0001]
	1940–1949	13.56	−0.085	−0.035	−0.017	18.6
			(0.012)	(0.012)	(0.011)	[0.0001]
High school graduate	1930–1939	0.77	−0.019	−0.020	−0.004	46.4
			(0.002)	(0.002)	(0.002)	[0.0001]
	1940–1949	0.86	−0.015	−0.012	−0.002	54.4
			(0.001)	(0.001)	(0.001)	[0.0001]
Years of educ. for high school graduates	1930–1939	13.99	−0.004	0.051	0.012	5.9
			(0.014)	(0.014)	(0.014)	[0.0006]
	1940–1949	14.28	0.005	0.043	−0.003	7.8
			(0.011)	(0.011)	(0.010)	[0.0017]
College graduate	1930–1939	0.24	−0.005	0.003	0.002	5.0
			(0.002)	(0.002)	(0.002)	[0.0021]
	1940–1949	0.30	−0.003	0.004	0.000	5.0
			(0.002)	(0.002)	(0.002)	[0.0018]
Completed master's degree	1930–1939	0.09	−0.001	0.002	−0.001	1.7
			(0.001)	(0.001)	(0.001)	[0.1599]
	1940–1949	0.11	0.000	0.004	0.001	3.9
			(0.001)	(0.001)	(0.001)	[0.0091]
Completed doctoral degree	1930–1939	0.03	0.002	0.003	0.000	2.9
			(0.001)	(0.001)	(0.001)	[0.0332]
	1940–1949	0.04	−0.002	0.001	−0.001	4.3
			(0.001)	(0.001)	(0.001)	[0.0050]

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TABLE II
PERCENTAGE OF AGE GROUP ENROLLED IN SCHOOL BY BIRTHDAY AND LEGAL DROPOUT AGE[a]

Date of birth	Type of state law[b]: School-leaving age: 16 (1)	School-leaving age: 17 or 18 (2)	Column (1) − (2)
	Percent enrolled April 1, 1960		
1. Jan 1–Mar 31, 1944 (age 16)	87.6 (0.6)	91.0 (0.9)	−3.4 (1.1)
2. Apr 1–Dec 31, 1944 (age 15)	92.1 (0.3)	91.6 (0.5)	0.5 (0.6)
3. Within-state diff. (row 1 − row 2)	−4.5 (0.7)	−0.6 (1.0)	−4.0 (1.2)

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  - no, if the effect works through education
  - yes, if it is a separate effect

4. Results

Interpreting the Results: Wald Estimate

OLS estimate:

$$\hat{\beta}_{OLS} = \frac{\Delta Y}{\Delta X}$$

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TABLE III
PANEL A: WALD ESTIMATES FOR 1970 CENSUS—MEN BORN 1920–1929[a]

	(1) Born in 1st quarter of year	(2) Born in 2nd, 3rd, or 4th quarter of year	(3) Difference (std. error) (1) − (2)
ln (wkly. wage)	5.1484	5.1574	−0.00898
			(0.00301)
Education	11.3996	11.5252	−0.1256
			(0.0155)
Wald est. of return to education			0.0715
			(0.0219)
OLS return to education[b]			0.0801
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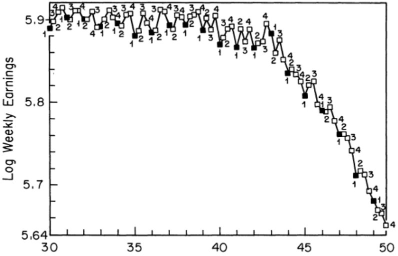
Does this look like an estimation method we already studied?

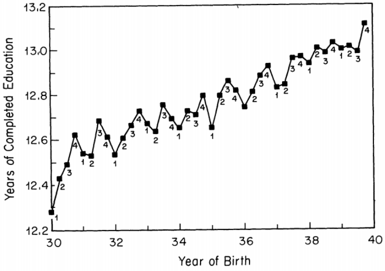
Visual Wald Estimate

Divide changes due to QOB in Figure 5 by changes due to QOB in Figure 1

Numerator: ΔY due to ΔZ



Denominator: ΔX due to ΔZ



Interpreting the Results: OLS and IV

- Interpret OLS coefficient
- Interpret IV coefficient

TABLE IV

OLS AND TSLS ESTIMATES OF THE RETURN TO EDUCATION FOR MEN BORN 1920–1929: 1970 CENSUS[a]

Independent variable	(1) OLS	(2) TSLS	(3) OLS	(4) TSLS	(5) OLS	(6) TSLS	(7) OLS
Years of education	0.0802 (0.0004)	0.0769 (0.0150)	0.0802 (0.0004)	0.1310 (0.0334)	0.0701 (0.0004)	0.0669 (0.0151)	0.0701 (0.0004)
Race (1 = black)	—	—	—	—	0.2980 (0.0043)	−0.3055 (0.0353)	−0.2980 (0.0043)
SMSA (1 = center city)	—	—	—	—	0.1343 (0.0026)	0.1362 (0.0092)	0.1343 (0.0026)
Married (1 = married)	—	—	—	—	0.2928 (0.0037)	0.2941 (0.0072)	0.2928 (0.0037)
9 Year-of-birth dummies	Yes	Yes	Yes	Yes	Yes	Yes	Yes
8 Region of residence dummies	No	No	No	No	Yes	Yes	Yes
Age	—	—	0.1446 (0.0676)	0.1409 (0.0704)	—	—	0.1162 (0.0652)
Age-squared	—	—	−0.0015 (0.0007)	−0.0014 (0.0008)	—	—	−0.0013 (0.0007)
χ^2 [dof]	—	36.0 [29]	—	25.6 [27]	—	34.2 [29]	—

5. A and K, 30 Years On

A and K, 30 Years On

- Compulsory schooling part still has traction
- Quarter of birth part not as much
- Subject to a scathing critique (BBJ, 1995), but method much copied
  - quarter of birth impacts school performance
  - health differences by quarter of birth
  - regional patterns in quarter of birth
- Next class: we'll discuss problems of using many and weak instruments

Next Lecture

- Read
  - the instrument paper to which you're assigned
  - trade if you'd like
  - skim the intro of the other
- Summary due next week if you're on the list