Microeconomics for Public Policy I
Fall 2020

## In Class Problems, Lecture 4

1. Complements and substitutes
For each of the following utility functions, explain whether $X$ and $Y$ are perfect complements, perfect substitutes, or some of both – and why.

(a). $U = U(X, Y) = XY$

Complementary: if you don't consume $X$, you can't get any utility at all. Same for $Y$. But you can also replace consumption of $X$ with consumption of $Y$ – though not always at the same amount.

(b). $U = U(X, Y) = X + Y$

These are substitutes. A unit of $X$ gives you the same utility as a unit of $Y$. You do not need $X$ to enjoy $Y$ and vice-versa.

(c). $U = U(X, Y) = X^{0.7}Y^{0.3}$

Same as the first one.

2. Utility maximization
Sarah gets utility from soda (S) and hotdogs (H). Her utility function is $U = S^{0.5}H^{0.5}$, $MU_S = 0.5\frac{H^{0.5}}{S^{0.5}}$, and $MU_H = 0.5\frac{S^{0.5}}{H^{0.5}}$. Sarah's income is \$12, and the prices of soda and hotdogs are \$2 and \$3, respectively.

a. Write the equation for Sarah's budget constraint

Sarah's budget constraint is

$$I = P_H H + P_S S,$$

where $I$ is income, $P_H$ is the price of hot dogs, $H$ is the number of hot dogs, $P_S$ is the price of soda and $S$ is the number of sodas.

Plugging in from what's given in the problem, we can write

$$12 = 3H + 2S.$$

b. Draw Sarah's budget constraint

The budget constraint is a negatively sloped line. If Sara spends all her income on hotdogs, she can buy 6 hotdogs ($H$-axis intercept). If she spends all her money on soda, she can buy 3 sodas ($S$-axis intercept). Now you have two points and can draw the line.

c. Write the marginal rate of substitution in terms of $H$ and $S$

We know that at equilibrium $MRS_{H,S} = \frac{P_H}{P_S}$. Given the terms of the problem, we can re-write this as

$$\frac{0.5H^{0.5}S^{-0.5}}{0.5S^{0.5}H^{-0.5}} = \frac{2}{3}.$$

We can simplify to

$$\frac{H}{S} = \frac{2}{3}.$$

d. What amount of sodas and hotdogs makes Sarah happiest, given her budget constraint? (Recall that you have two equations and two unknowns.)

Recall from the first part that we also know that $12 = 2S + 3H$. We learned above that $H = \frac{2}{3}S$.

So now we have two equations and two unknowns. We can therefore write

$$\begin{aligned} 12 &= 2S + 3\frac{2}{3}S \\ 12 &= 2S + 2S = 4S \\ S &= 3 \end{aligned}$$

If $S = 3$, then $12 = 2 * 3 + 3H$, which means that $6 = 3H$, or $H = 2$. And Sarah has spent all her money.

3. GLS Chapter 4, Question 8 (Second edition: question 9)

We are given that $U = 4XY$ and $MU_X = 4Y$, and $MU_Y = 4X$.

(a) Suppose that $Y = 3$. Calculate utility for $X = \{2, 3, 10, 11\}$. I did this by making a table:

| $X$ | $Y$ | $U = 4XY$ |
|---|---|---|
| 2 | 3 | $4(2)3 = 24$ |
| 3 | 3 | $4(3)3 = 36$ |
| 10 | 3 | $4(10)3 = 120$ |
| 11 | 3 | $4(11)3 = 132$ |

Notice that an additional unit of $X$ yields $36-24 = 12$ additional utils from $X = 2$ to $X = 3$. Note also that from $X = 10$ to $X = 11$, the additional unit of $X$ yields $132 - 120 = 12$ additional utils. Thus, marginal utility is not decreasing in $X$.